PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Origin of Planetary Rings

S. F. Dermott

Phil. Trans. R. Soc. Lond. A 1981 **303**, 261-279
doi: 10.1098/rsta.1981.0200

Phil. Trans. R. Soc. Lond. A **303**, 261–279 (1981)
Printed in Great Britain

The origin of planetary rings

By S. F. Dermott
Center for Radiophysics and Space Research, Space Sciences Building, Cornell University, Ithaca, New York 14853, *U.S.A.*

[Plates 1–4]

Recent spacecraft and ground-based observations have revealed the presence of narrow rings encircling the planets Jupiter, Saturn and Uranus. The Jovian ring is known to contain at least two small, dark, satellites of diameter between 20 and 40 km in its outer edge. The structure of the Saturnian F ring has been resolved by Voyager 1 and appears to be determined by the action of two small neighbouring satellites which were also imaged by the spacecraft. All nine Uranian rings are extremely narrow and some are appreciably eccentric. The outer ε ring has very sharp edges and its radial width increases from 20 km at pericentre to 100 km at apocentre. This marked variation in width is also characteristic of the Uranian α and β rings and of a narrow ring in the Saturnian system. The structure of the Uranian η ring is complex and may be similar to that of the Saturnian F ring. The resolution of the numerous, but well defined dynamical problems posed by these narrow rings must precede any discussion of the origin of rings. Two co-orbital Saturnian satellites that appear to move in horseshoe orbits have been discovered. The stability of these orbits and the origin of these and other co-orbital satellites are discussed.

1. Introduction

The rings that exist round Jupiter, Saturn and Uranus are of importance not simply because of the interesting dynamical problems that they pose, but because it is probable that the processes that have been invoked to account for their structure had a role in planet and satellite formation. In a sense, ring systems, and here I include the asteroid ring, are examples of arrested or delayed growth. They probably afford our best opportunity of studying, in their full complexity, some of the accretion mechanisms that operated in the early Solar System.

Before March 1977 the only planetary ring known to astronomers was that of Saturn. Ground-based observations of that ring produced data on the size and composition of the ring particles, on the azimuthal variations of ring brightness and on the gross structure of the ring system. Cassini's division and Encke's gap and other structural features were tentatively ascribed to resonant gravitational interactions between the orbits of the ring particles and those of Mimas and other satellites, for example, Titan. The formation of the rings was ascribed either to the break-up of a comet or a satellite that was brought inside Roche's limit or to some primordial accretion process. However, because of the lack of any stringent observational tests to differentiate between these choices, these theories remain speculative.

The discovery in March 1977, by Elliot *et al.* (1977) and other groups, of the narrow Uranian rings, the discovery in March 1979 by Voyager 1 of the comparatively narrow Jovian ring, and the discovery in September 1979 by Pioneer 11 of the Saturnian F ring produced a new focus of interest for planetary science. The Uranian rings in particular, some of which are known to be eccentric and of variable width, present a nicely posed, highly constrained, dynamical problem. That some of the models that have been proposed to account for the existence of narrow, eccentric rings will be tested and perhaps confirmed in the very near future is a direct result of and a testimony to the exploration of the outer Solar System by N.A.S.A. spacecraft.

Before the Saturn encounter of Voyager 1 in November 1980, the contrast between the broad Saturnian rings and the narrow Uranian rings appeared striking. Although the division of the system into the A, B, C and D rings is still meaningful and significant, we now know that these broad rings consist of a very large number of narrow ringlets. Voyager 1 confirmed that the F ring is narrow and also discovered two narrow, eccentric rings. It is appropriate therefore that this brief review is confined chiefly to a discussion of the origin and dynamics of narrow, eccentric rings.

2. Ring data

The gross structures of the Jovian, Saturnian and Uranian rings are described in tables 1, 2 and 3. The locations of the chief ring features with respect to the Roche zones of the planets, as defined by Dermott *et al.* (1979), are shown in figure 1.

The Jovian ring is optically thin, comparatively narrow and appears to contain little structure. Note, however, that, because of smear motion, features smaller than *ca.* 700 km cannot be resolved in the Voyager images (Jewitt & Danielson 1981). The visible ring particles are small (*ca.* 2 μm) and have short lifetimes ($\lesssim 10^2$–10^3 years) limited by erosion due to sputtering and meteoroid impacts. Thus, they must be replenished, most probably by some source within the rings (see review by Burns *et al.* (1981)). This fact alone suggests the existence of a number of small satellites within the rings (Dermott *et al.* 1980).

Table 1. Jupiter data

(From Jewitt & Danielson 1981.)

ring, feature or satellite	location R_J†	width, diameter km	optical depth
faint ring, inner radius	1	—	7×10^{-6}
outer radius	1.72 ± 0.01	—	—
transition zone	1.72	1500	—
bright ring, inner radius	1.72 ± 0.01	6000	3×10^{-5}
outer radius	1.81 ± 0.01‡	—	—
bright annulus§	1.79	700	—
halo	1 to 1.81	10000‖	—
1979 J1	1.79	30 to 40	—
1979 J3¶	1.79	*ca.* 40	—

† Radius of Jupiter, R_J, is 71400 km.
‡ There is some evidence that the ring extends to 1.84 R_J (Jewitt & Danielson 1981).
§ Annulus is 10% brighter than surrounding ring.
‖ Height above ring plane.
¶ Data from Voyager Bulletin, Mission Status Report no. 52 (27 August 1980).

Close examination of the Voyager images has revealed the presence of at least one, and probably two, small, dark satellites close to the outer edge of the ring as seen by back-scattering of sunlight (see table 1 for data). Because of smear motion, the exact location of these satellites is uncertain by *ca.* 700 km (Jewitt & Danielson 1981). Other satellites may exist within the rings but according to Jewitt & Danielson (1981) their dimensions must be $\lesssim$ 1 km. However, I note that the latter statement was made before the discovery of 1979 J3. It would be interesting to know if a small satellite defines the inner edge of the ring at 1.72 planetary radii. A search has

TABLE 2. SATURN DATA

ring, feature	location† R_S‡	width† km	optical depth
D ring	1 to 1.21	—	—
C ring	1.21 to 1.53	—	*ca.* 0.1
B ring	1.53 to 1.97	—	*ca.* 1
Cassini division	1.97 to 2.03	4200	*ca.* 0.06
A ring	2.03 to 2.26	—	*ca.* 0.5
Encke division	2.21	876 ± 35	—
F ring	2.32	*ca.* 30	*ca.* 1
G ring	2.8	—	—
E ring	3.5 to 4.91	—	10^{-6}
co-orbital satellites	2.51	—	—

† Locations and widths are from Voyager 1 data (Voyager Bulletin, Mission Status Report no. 59, 21 November 1980) and from the Poineer 11 data of Gehrels *et al.* (1980).
‡ Radius of Saturn, R_S, is 60000 km.

TABLE 3. URANUS DATA

(From Elliot *et al.* (1981*a*, *b*), except where marked otherwise.)

ring	semi-major axis, *a* km	10^3 eccentricity, *e*	mean width, $2\Delta a$ km	range of eccentricities $2\Delta e \times 10^4$	eccentricity gradient $a\Delta e/\Delta a$
6	41865.5 ± 32.1	1.36 ± 0.07	—	—	—
5	42272.0 ± 32.2	1.77 ± 0.06	—	—	—
4	42600.1 ± 32.3	1.24 ± 0.09	—	—	—
α	44752.3 ± 32.4	0.72 ± 0.03	8.4†	0.64†	≳ 0.34†
β	45695.6 ± 32.4	0.45 ± 0.03	8.4†	0.79†	≳ 0.43†
η	47208.9 ± 32.5	(0.03 ± 0.04)	—	—	—
γ	47657.3 ± 32.5	(0.04 ± 0.04)	—	—	—
δ	48333.9 ± 32.6	0.54 ± 0.035	—	—	—
ε	51181.7 ± 33.3	7.93 ± 0.04	57.0 ± 0.5	7.06 ± 0.13	0.634 ± 0.013

† Data from P. Nicholson (private communication, 1980).

been made without success, but the Voyager images cover only two-thirds of the ring and it is still possible that such a satellite exists (Jewitt & Danielson 1981).

Before the discovery of the Jovian ring, theories of the structure of the Uranian rings had been advanced that advocate the gravitational action of unseen satellites on the orbits of the ring particles (Goldreich & Tremaine 1979; Dermott *et al.* 1979). The association of small satellites with the Jovian ring lent credibility to these theories and this has been enhanced by the results of the Voyager 1 Saturn encounter. The gravitational action of small satellites on the orbits of ring particles is the dominant theme of this review.

Near-infrared reflectance spectra show that the Saturnian ring particles have surfaces of water ice (Pilcher *et al.* 1970). The particles are excellent reflectors of centimetre radar waves and thus must have typical sizes ≳ 1 cm (Goldstein & Morris 1973). If the particles are made of water ice, then the fact that they are very weak radio emitters at centimetre wavelengths limits their size to < 10 m (Berge & Muhleman 1973; Briggs 1974). Saturn's rings and ring material, in contrast to those of both Jupiter and Uranus, are bright. The Saturnian C ring is darker than the A and B rings and there is some evidence that the C ring material is different from that in the A and B rings (Dilley 1980; and early Voyager 1 reports, 1980).

The passage of the Earth through the Saturnian ring plane in March 1980 resulted in a number of discoveries and the existence of the E ring previously discovered by Feibelman (1967) was

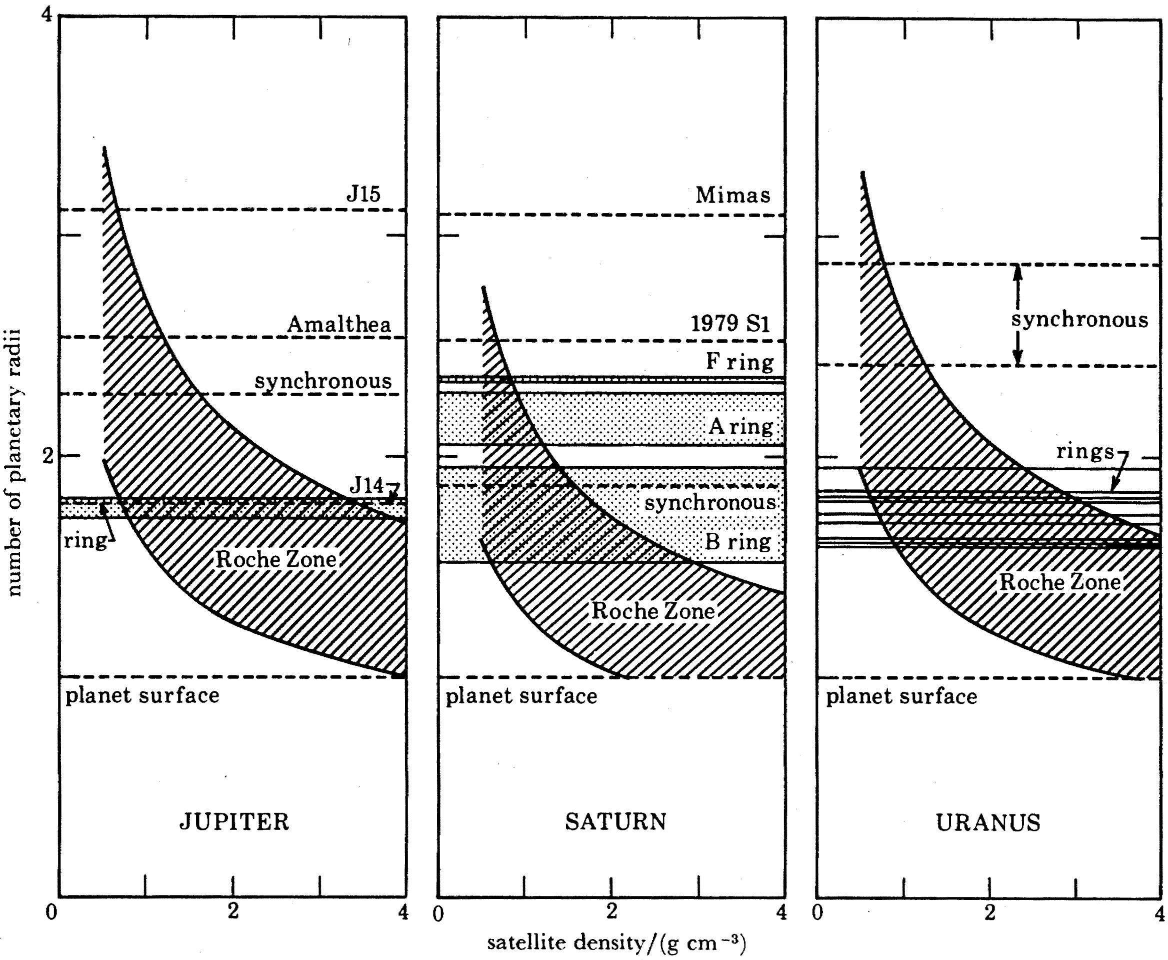


FIGURE 1. Locations of the chief ring features with respect to the Roche zones and the synchronous orbits of the planets. In the Roche zone a satellite of a certain density can coexist with a ring of particles of the same density.

confirmed (Baum *et al.* 1980; Brahic *et al.* 1980; Dollfus & Brunier 1980; Lamy & Mauron 1980; Reitsema *et al.* 1980*a*; Terrile & Tokanaga 1980). This tenuous ring is not without structure. Baum *et al.* (1980) and others have found that the radial distribution of ring material peaks at the orbit of Enceladus suggesting to them that the E ring may be of recent origin. Brahic *et al.* (1980) found a further peak in brightness at 5.7 radii. The reflectivity curve of the E ring suggests to Terrile & Tokanaga (1980) that the ring particles may be small ($< 5\,\mu m$), in which case they would be short-lived and one would have to find a source of replenishment. Even more intriguing are the observations by Dollfus & Brunier (1980) and by Lamy & Mauron (1980) that the radial variation of E ring brightness varies with time. This suggests that the E ring is azimuthally asymmetric and may consist, at least in part, of narrow arcs of material bound to the Lagrangian equilibrium points of some of the satellites, in particular, Enceladus. However, no such arcs have been seen by Voyager 1.

With, perhaps, the exception of the F ring of Saturn, the rings of Uranus present the most inviting challenge to dynamicists. For these rings it is possible to list a large number of significant features that any theory of the rings ought to be able to account for.

All the radii of the confirmed nine rings lie in a comparatively narrow spread $\lesssim 10000$ km.

The innermost ring is 16000 km above the surface of the planet and the outermost ring is 7800° km lower than the orbit of Miranda. All the rings, except the ε ring, have apparent mean widths of the same order: *ca.* 3–8 km (Nicholson *et al.* 1978; P. D. Nicholson, personal communication, 1980; Elliot *et al.* 1981*b*). Note, however, that most of the narrow rings (mean widths < 8 km) have not yet been resolved. All of the rings are apparently optically thick (optical depths $\tau \gtrsim 1$ (Nicholson *et al.* 1978)) and the ring material is very dark. Smith (1977) finds that at 886 nm (a deep methane absorption band in which the reflectivity of the planet is particularly low, *ca.* 0.012) the albedo is $\lesssim 0.02$. In May 1978 the rings were successfully mapped at 2.2 μm and at this wavelength their albedo is *ca.* 0.05 (Goldreich 1979). Thus, the albedo is much closer to that of carbonaceous chondritic material than to that of the ice-coated particles surrounding Saturn (Smith 1977).

The sides of some of the Uranian ring occultation profiles are remarkably steep. For the γ and ε ring profiles shown in figures 2 and 3, there is no suggestion of a gradual change in opacity with orbital radius. The ε, α and η (see figure 16) rings are in fact more opaque at their edges than at their centres (Elliot *et al.* 1981*b*). The shapes of the ε profiles are remarkably similar even though their overall width varies from 20 to 100 km (Nicholson *et al.* 1978).

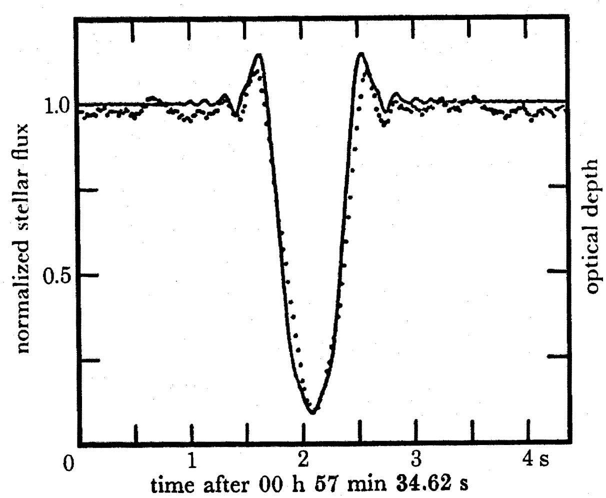


FIGURE 2. Occultation profile of the Uranian γ ring (Elliot *et al.* 1981*b*). The points represent 20 ms averages of the data and the solid line is a model profile for an occultation of a star 0.0018″ in diameter by an opaque ring 3.4 km wide. The peaks at the edges of the profile are due to Fresnel diffraction by the abrupt boundaries of the ring.

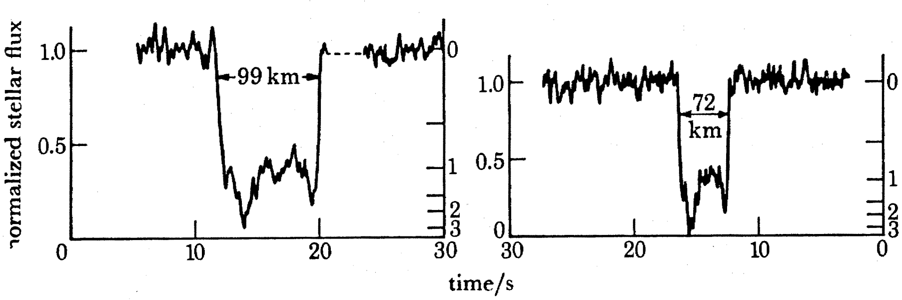


FIGURE 3. Comparison of two wide profiles of the Uranian ε ring, obtained on 10 March 1977 (Millis *et al.* 1977) and 10 April 1978 (Nicholson *et al.* 1978). Differing width scales are due to different projected velocities of Uranus relative to the two stars.

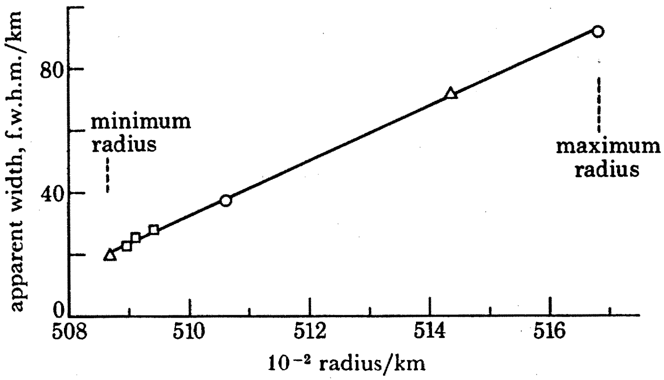


FIGURE 4. Radial width versus radius for the Uranian ε ring (Elliot *et al.* 1981*a*). The apparent radial width (f.w.h.m.) is plotted against the model radius with use of data from three separate occultations: ○, 10 March 1977; △, 10 March 1978; □, 20 March 1980.

Most of the Uranian rings are clearly eccentric, but the orbits of the particles in any one ring have common pericentres. This precise apse alignment is maintained despite the fact that differential precession enforced by the oblateness of the planet acts against alignment. The widths of the ε, β and probably the α rings (P. D. Nicholson, personal communication, 1980) at any one point increase linearly with the radial distance of that point from the centre of the planet (see figure 4), and the ratios of the maximum and minimum widths of these rings are substantially greater than unity (Nicholson *et al.* 1978). Voyager 1 has now discovered that two of the Saturnian rings are also clearly eccentric and at least one of these is of variable width (see figures 13 and 14). In all cases, the rings are widest at apocentre.

3. Confinement of narrow rings

Goldreich & Tremaine (1979) have pointed out that unconstrained rings spread for two reasons. (i) Collisions between particles, for which the characteristic diffusion time t_d is comparable to the time a particle needs to random walk across the ring (Brahic 1977; Goldreich & Tremaine 1978*a*),

$$t_d \approx \frac{1}{\tau' n}\left(\frac{W}{d}\right)^2, \tag{1}$$

where W is the mean ring width and n is the mean motion of the ring particles; τ' is not simply the optical depth of the ring (this is always determined by the number density of the smaller particles), but the optical depth that the ring would have if all particles other than those larger particles of characteristic radius d, that determine the collisional spreading rate, were removed. For rings located at *ca.* 2 planetary radii,

$$t_d \approx \frac{10^9}{\tau'}\left(\frac{\text{cm}}{d}\right)^2\left(\frac{W}{10\,\text{km}}\right)^2 \quad \text{years.} \tag{2}$$

(ii) Poynting–Robertson light drag produces spreading on a time scale

$$t_{pr} \approx \frac{\pi \rho d a_p^2 c^2 W}{r L_\odot}, \tag{3}$$

where ρ is the density of the ring material, a_p is the mean orbital radius of the planet, c is the velocity of light, $L_\odot$ is the solar luminosity and r is the radius of the ring. For the Uranian rings,

$$t_{pr} \approx 10^6\left(\frac{d}{\text{cm}}\right)\left(\frac{W}{5\,\text{km}}\right) \quad \text{years.} \tag{4}$$

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Since t_d increases and t_{pr} decreases as d increases, and vice versa, it follows from the above that, regardless of particle size, narrow rings could not survive for more than *ca.* 10^7 years. Therefore, if the rings are not young, then they must be confined (Goldreich & Tremaine 1979).

4. The Goldreich–Tremaine theory

Goldreich & Tremaine (1979) consider that particles in the narrow rings of Uranus are confined by tidal torques exerted on them by small, unseen satellites. With the recent discovery by Voyager 1 of two small satellites that bound the narrow F ring of Saturn, their theory would appear to be confirmed. However, the structure of the F ring is complex and quite unlike that of the ε ring of Uranus which inspired their theory. Some of the images of the F ring show two bright rings which *appear* to be intertwined or braided. These two rings are closely associated with a third, but this is very diffuse (see figure 14, plate 3).

The tidal torque exerted on a narrow ring of particles of total mass m_r by a nearby satellite of mass m can be estimated from the scattering angle δ in figure 5:

$$\delta = 2Gm/xU^2, \tag{5}$$

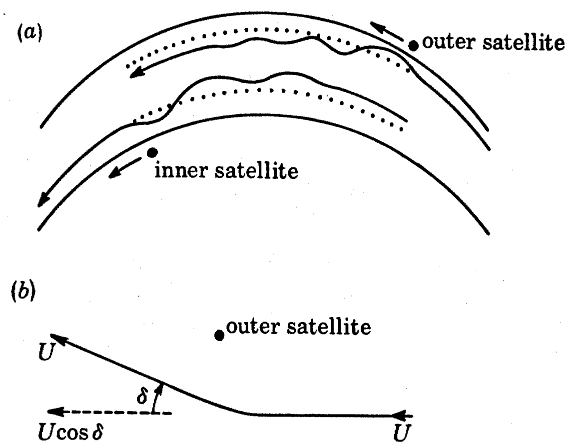


Figure 5. Constraint of a narrow ring by two small satellites. (*a*) Angular velocity decreases with increasing distance from the planet. Thus ring particles leading the outer satellite and lagging the inner satellite will have eccentric orbits. The interaction of these two waves, which have equal amplitudes but different wavelengths, could result in some novel phenomena. (*b*) The amplitude of the waves can be estimated from the scattering angle δ.

where the relative velocity U is given by

$$U = \pm \tfrac{3}{2}nx, \tag{6}$$

G is the gravitational constant and x is the separation of the satellite and the ring. Since the tangential component of the relative velocity is reduced from U to $U\cos\delta$, angular momentum is exchanged with the net result that the ring experiences a torque

$$T = \pm \frac{4}{9\pi}\left(\frac{Gm}{nx^2}\right)^2 m_r. \tag{7}$$

The sign of the torque depends on whether the satellite is on the inside or the outside of the ring and is such that the satellites repel the ring particles (Lin & Papaloizou 1979).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

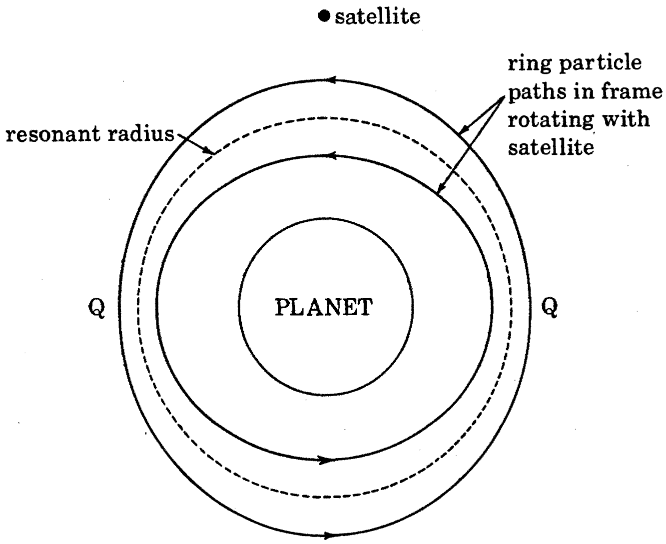


FIGURE 6. Ring particle paths in a frame rotating with the perturbing satellite. Resonant interactions at radii in the disk where the ratio of the satellite and particle mean motions are close to $p/(p+1)$, where p is an integer, generate density enhancements at the points Q.

Since $x \ll r$, the gravitational force on a ring particle due to a satellite is only effective at close encounters. During encounter, a particle initially moving in a circular orbit acquires a radial velocity $U \sin \delta$ and thereafter moves in a Keplerian ellipse of eccentricity

$$e \approx \frac{U \sin \delta}{nr} = \frac{4}{3}\frac{m}{M}\left(\frac{r}{x}\right)^2. \tag{8}$$

In a frame corotating with the perturbing satellite, all particles initially moving in circular orbits must follow identical paths after encounter. Thus each satellite generates a standing wave of amplitude

$$A \approx er \tag{9}$$

and wavelength

$$l = 2\pi U/n = 3\pi x \tag{10}$$

(see figure 5). In the inertial frame, each particle moves in an independent Keplerian ellipse, but the pericentres of these elliptical orbits and the phases of the particles on the orbits are such that the locus of the particles is a sinusoidal wave that moves through the ring with the angular velocity of the perturbing satellite.

The damping of these waves, by collisions, results in a net exchange of angular momentum between the satellite and the ring particles. If a ring is bounded by two satellites, then its width will be reduced until the confining torques just counteract the tendency of the ring to spread. The equilibrium width of the ring is

$$W \approx \left(\frac{3\pi\tau'}{4}\right)^{\frac{1}{2}}\left(\frac{x}{r}\right)^{\frac{5}{2}}\left(\frac{M}{m}\right) d. \tag{11}$$

For the F ring of Saturn

$$d \approx \frac{9}{\tau'^{\frac{1}{2}}}\left(\frac{30\,\text{km}}{W}\right)\left(\frac{m/M}{2\times10^{-9}}\right)\left(\frac{1000\,\text{km}}{x}\right)^{\frac{5}{2}} \text{metres} \tag{12}$$

($m/M \approx 2\times10^{-9}$ corresponds to a satellite of diameter 130 km and density 1 g cm^{-3}). The ring must have a substantial population of particles with sizes $\gtrsim$ 9 m.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

When the ring is in equilibrium, the torques due to the satellites are equal and opposite, hence, from equation (7),

$$\frac{m_o}{x_o^2} = \frac{m_i}{x_i^2}, \tag{13}$$

where subscripts o and i refer to the outer and inner satellite respectively. Verification of the above condition for the F ring of Saturn would serve as a test of the Goldreich–Tremaine theory. For the purposes of this paper, I assume that the satellites and the ring particles move in coplanar, circular orbits, but, of course, in any test one would have to take account of the observed ring–satellite orbit configuration.

If the tidal torques on the ring are equal, then it follows from equations (8) and (13) that the amplitudes of the two waves are equal. For the F ring of Saturn

$$A \approx 7\left(\frac{m/M}{2\times 10^{-9}}\right)\left(\frac{1000\,\text{km}}{x}\right)^2 \text{km} \tag{14}$$

and may be comparable with the width of the ring ($W \lesssim 30$ km). The interaction of these two waves, which probably have quite different wavelengths, could produce some novel phenomena: beats, for example. If $W \ll x$, then the perturbations of the ring particle orbits will be highly coherent and although particle collisions will act to damp the waves the process may be slow. If the waves can survive from one encounter to the next, then one must consider the possibility of resonance. This will occur if the ring circumference is an integral number of wavelengths, that is, if

$$2r/3x = p+1, \tag{15}$$

where p is an integer ($\geqslant 1$). Consecutive perturbations will then be in phase and a wave of amplitude significantly greater than A may result. I have described (Dermott 1981) how resonant gravitational interactions in a narrow ring could excite a wave pattern of $p+1$ equally spaced loops and this could account for the braided appearance of the F ring. Since the ring is appreciably perturbed by more than one satellite it is possible that such loops are transient features that are continually excited by resonant interactions but then destroyed by destructive interference of waves and by shock phenomena produced by close packing of ring particles.

Resonant gravitational interactions have also been invoked by Goldreich & Tremaine (1978*b*) to account for gap formation. For particles close to the exact resonance, the resonance condition is

$$pn-(p+1)\,n'+\dot{\tilde{\omega}} = 0, \tag{16}$$

where $\tilde{\omega}$ is the pericentre of a ring particle orbit and n' is the mean motion of the perturbing satellite ($n' < n$). If the particles are locked in resonance, and the forced eccentricities are small, then the pericentres of the ring particle orbits are not aligned, rather $\tilde{\omega}$, $\dot{\tilde{\omega}}$ and the mean longitude, or phase, are such that conjunctions of a particle and the satellite always occur at an apse of that particle's orbit (Greenberg 1973). The magnitude of the forced eccentricity, in the absence of damping, is given by

$$e = \left|\frac{1}{2}\,\frac{(m/M)f(p)\,n'}{(p+1)\,n'-pn}\right|. \tag{17}$$

Thus e increases markedly as the exact resonance is approached. At the exact resonance, the phase of the response changes by 180°. Similar behaviour is observed in any driven harmonic oscillator. For particles outside the exact resonance, conjunction always occurs at apocentre, whereas for particles on the inside of the exact resonance, conjunction always occurs at pericentre. Thus, the satellite excites a wave pattern of equally spaced loops.

In figure 6 I show these loops for the case $p = 1$. This could represent the action of Mimas on particle orbits near the 2:1 resonance in Saturn's rings, that is, near Cassini's division. Goldreich & Tremaine (1978*b*) consider that resonant gravitational interactions in a broad disk of particles produce density enhancements at the points marked Q in figure 6. These density enhancements move through the disk with the angular velocity of the perturbing satellite and their gravitational action on the orbits of the particles in the disk produces a spiral density wave. This wave carries negative energy and angular momentum and propagates outward towards the satellite. The wave is damped by particle collisions, and those particles involved in the damping lose energy and angular momentum and move inwards the planet. Consequently, just *outside* the exact resonant radius a broad gap is opened up.

This theory would appear to account for some of the major ring features. The *inner* edge of Cassini's division is close to the 2:1 Mimas resonance and spiral density waves have been detected by Voyager 1 both in Cassini's division and in the A ring. However, the revelation by Voyager 1 of $\gtrsim$ 1000 gaps in the rings (see figures 12–14, plates 1–3) makes it likely that resonant gravitational interactions involving satellites outside the rings will prove insufficient to account for all the observed structure. Once again one may have to invoke the action of unseen satellites orbiting within the rings.

One important consequence of the Goldreich–Tremaine theory of gap formation is that regions just inside the strongest resonances may have been preferred sites of satellite formation. It is remarkable that the recently discovered co-orbital satellites of Saturn (orbital radii 2.51 planetary radii) lie just inside the 4:3 Mimas resonance (at 2.55 planetary radii) and the F ring with its attendant satellites (orbital radius 2.32 planetary radii) lies just inside the 3:2 Mimas resonance (at 2.36 planetary radii). If these satellites are significantly close to resonance, but not trapped in exact resonance, then they lend support to the hypothesis that the marked preference for orbit–orbit resonance in the Solar System is primordial rather than the result of long-term orbital evolution due to, say, tidal dissipation.

5. The horseshoe-orbit model

The extremely narrow widths and sharply defined edges of the Uranian rings prompted Dermott *et al.* (1979) to suggest that some kind of critical phenomenon is responsible and they proposed that each narrow ring contains a small satellite which maintains solid particles in stable, horseshoe orbits about its Lagrangian equilibrium points (see figure 7). In their model, loose solid particles leave the satellite surface and enter orbits closely similar to that of the satellite, which can be either circular or eccentric. It is the gravitational force of the satellite in a 1:1 resonance with the co-orbital ring particles that then provides the critical phenomemon needed to define a narrow, sharp-edged asymmetric ring.

The path of a ring particle when the orbit of the ring satellite is circular can be studied by considering the Jacobi integral (Brown 1911). For motion in horseshoe orbits it is convenient to write the Jacobi constant C as

$$C = 3 + \alpha(m/M)^{\frac{2}{3}}, \tag{18}$$

where α is a constant $\leqslant 3^{\frac{4}{3}}$. If we write

$$a = a_s + \Delta a, \tag{19}$$

where a and a_s are the semi-major axes of a ring particle and the ring satellite respectively, then,

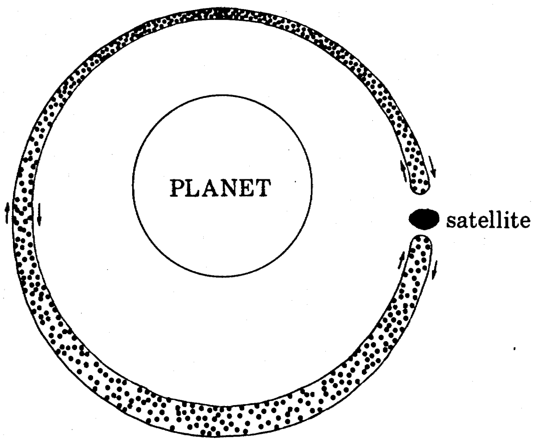


FIGURE 7. Dermott *et al.* (1979) suggested that each narrow ring contains a small satellite that maintains particles in stable, horseshoe orbits about its Lagrangian equilibrium points. The orbit of the ring satellite can be either circular or eccentric. It is the gravitational force of the satellite in a resonance with the ring particles that provides the critical phenomenon needed to define a narrow, sharp-edged, asymmetric ring.

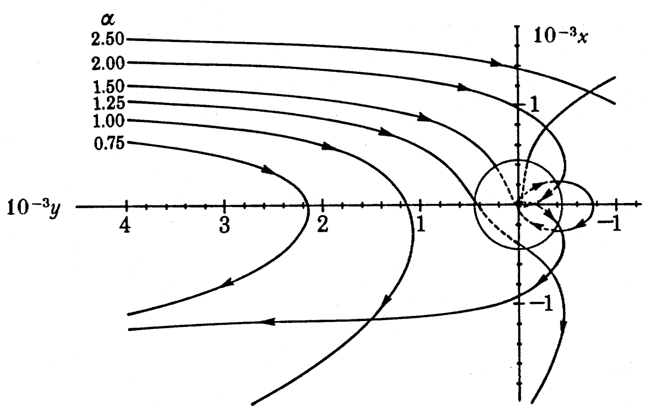


FIGURE 8. Particle paths in a frame rotating with the ring satellite. The ratio m of the satellite and planet masses is 10^{-9} and the satellite orbit is circular. The satellite radius corresponds to that of a satellite of mean density 2 g cm^{-3} at a distance of 2 planetary radii from Uranus. For high ($\gtrsim 1$) values of α (an impact parameter), the ring particles are either captured or scattered by the satellite, but for small values of α motion in horseshoe orbits is possible.

using Hill's equations (see Brouwer & Clemence 1961, p. 337), we can show that, for motion of a ring particle in a circle,

$$\Delta a = 2\left(\frac{\alpha}{3}\right)^{\frac{1}{2}}\left(\frac{m}{M}\right)^{\frac{1}{3}} a_{s} \tag{20}$$

and that the distance of closest approach of the particle to the satellite, y, is given by

$$y = \frac{2}{\alpha}\left(\frac{m}{M}\right)^{\frac{1}{3}} a_{s}, \tag{21}$$

(Dermott & Murray 1980).

Dermott *et al.* (1980) have found by numerical integration of particular cases that the type of path followed by a particle depends on the value of α. Figure 8 shows some of their results for $m/M = 10^{-9}$. For high values of $\alpha (\gtrsim 1)$ the particles either strike the ring satellite or are scattered, but for small values of α (< 1) the particles are repelled by the satellite and motion in horseshoe orbits is possible (for a simple discussion of the dynamics involved see Dermott *et al.* (1979)). The

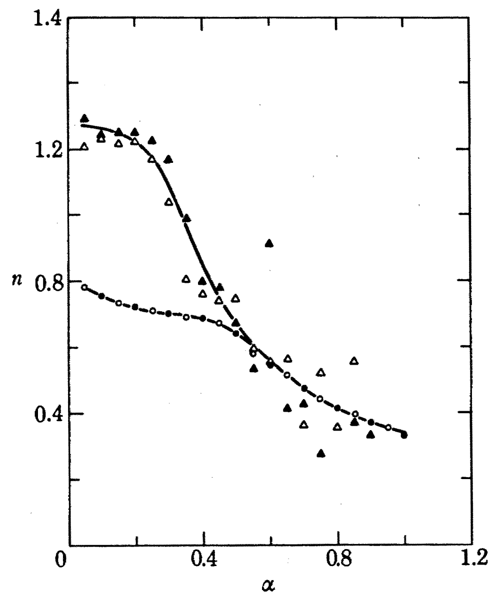


FIGURE 9. Power n is a measure of the symmetry of the path about the line $a = 1$ (see equation (22) and figures 8 and 10). The circular points refer to changes in the semi-major axis a of the ring particle orbit after a single encounter with the ring satellite: ●, the circular orbit case ($e_s = 0$, where e_s is the eccentricity of the ring-satellite orbit); ○, the elliptical orbit case ($e_s = 0.01$). Triangles indicate the total change in a after two consecutive encounters: ▲, $e_s = 0$; △, $e_s = 0.01$.

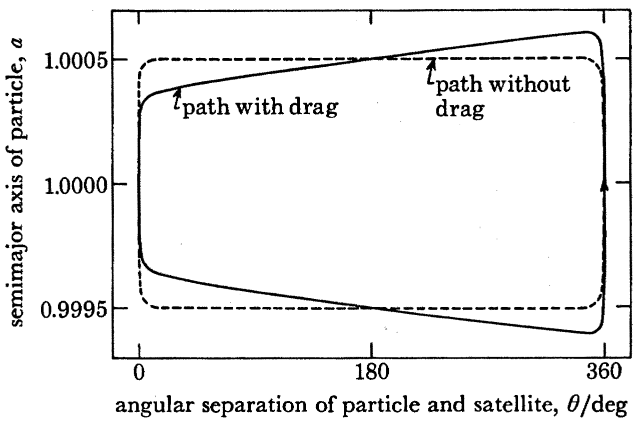


FIGURE 10. Path of a particle moving in a horseshoe orbit around a ring satellite of mass ratio $m = 10^{-9}$ (circular orbit case). The dashed line refers to the particle path in the absence of drag and the solid line shows the effect of an external drag force. Since $\alpha < 0.2$, both paths are highly symmetric about the line $a = 1$.

nature of the path changes dramatically as α is reduced and for very small values of α ($\lesssim 0.2$) the path is almost perfectly symmetric about the y axis. Dermott *et al.* (1980) suggested that it is this symmetry that accounts for the peculiar stability of very narrow rings.

If one writes

$$|\Delta a_0| - |\Delta a_j| = \pm (m/M)^n \quad (j = 1, 2), \tag{22}$$

where the subscript refers to the number of consecutive encounters with the ring satellite that the particle has experienced, then we find that n increases to values $\gtrsim 0.7$ ($j = 1$) and *ca.* 1.2. ($j = 2$) as α decreases to $\lesssim 0.2$ (see figure 10). Thus, for small values of α the orbits are near-periodic and Δa_0 and Δa_2 are equal and opposite to order *ca.* m/M. This symmetry is not a property of the

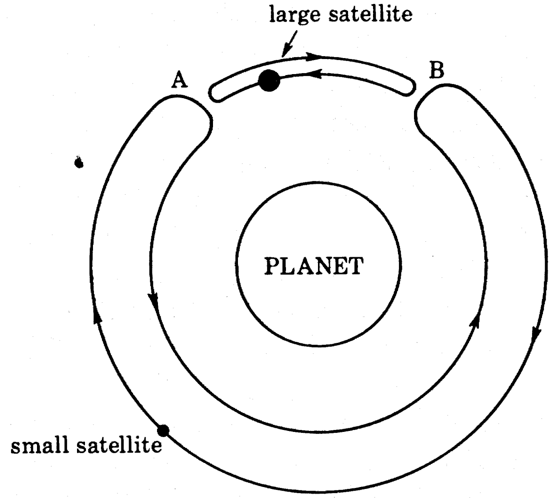


FIGURE 11. Two inner satellites of Saturn are found in the same orbit with a radius of 151400 km (Smith *et al.* 1980). Their rate of approach appears to be constant at 0.25°/day. This suggests that both satellites are moving in horseshoe orbits. The schematic paths shown here are drawn in a frame rotating with the average mean motion of both satellites.

circular orbit case alone. Figure 10 shows that there is no substantial difference between the circular and the eccentric orbit cases.

The effect on the orbit of a ring particle of an external force due, for example, to Poynting–Robertson light drag can now be understood. If α is small, then Δa_0 and Δa_2 are always equal and opposite: drag forces have little influence on this phenomenon. Therefore, if the ring is very narrow and the magnitudes of the drag forces acting on the particle on the inside and on the outside of its horseshoe path are not too different, then the orbital decay achieved in one half of the path is, in effect, cancelled by that achieved in the other (see figure 11). Since the drag force extracts angular momentum from the ring system, some orbital decay must, of course, occur, but the orbits of the ring particles and the ring satellite decay together at some rate r times less than than that of an unconstrained ring particle:

$$r \approx \frac{m}{m+m_r}, \tag{23}$$

where m_r is the total mass of the ring particles. Thus, even if a ring consists of very small particles ($d \ll 0.1$ cm), since $m \gg m_r$, Poynting–Robertson light drag acting over times comparable with the age of the Solar System would not result in significant orbital decay or ring spreading.

However, the lifetimes of particles in horseshoe orbits are finite. If equation (22) were sufficient to describe the effects of particle and satellite encounters, then we would expect particles to be lost from a ring due to a random walk of the quantity $|\Delta a_0| - |\Delta a_2|$. This would occur on a timescale

$$\Gamma \lesssim T/(m/M)^{\frac{4}{3}}, \tag{24}$$

where T is the orbital period of the ring satellite. It may be of significance that Γ increases as the mass of the ring satellite decreases: only very narrow rings may have lifetimes greater than the age of the Solar System. (Note, $\alpha \approx 0.2$ probably defines the edge of the ring, and it follows that for rings at *ca.* 2 planetary radii the width of the ring is approximately equal to the diameter of the co-orbital satellite.)

For the Jupiter–Sun system, $m/M \approx 10^{-3}$ and $T = 12$ years; hence $\Gamma \approx 10^6$ years and the lifetime of a ring co-orbital with Jupiter would be short. It is probably significant that none of the Trojan asteroids are observed to have horseshoe paths. For the Dione–Saturn system, $m/M \approx 2 \times 10^{-6}$ and $T \approx 10^{-2}$ year; hence $\Gamma \approx 2 \times 10^{7}$ years. Again, it is probably significant that the co-orbital satellite of Dione, Dione B, moves in a tadpole orbit (Lecacheux *et al.* 1980; Reitsema *et al.* 1980*b*). Dermott *et al.* (1979) have also pointed out that a tadpole rather than a horseshoe path is more likely unless $(m/M)^{\frac{1}{3}} \ll 1$: Jupiter and Dione do not satisfy this criterion. However, for the newly discovered co-orbital satellites of Saturn, m/M for the larger satellite of the pair is *ca.* 4×10^{-9} and $T \approx 2 \times 10^{-3}$ year; hence $\Gamma \approx 2 \times 10^{11}$ years and the horseshoe orbit configuration is both more likely than the tadpole and long-lived. Of course, the confirmed existence of co-orbital satellites is the best testimony to the stability of the orbits that I am describing.

A schematic diagram of the horseshoe paths of the Saturnian co-orbital satellites, S10 and S11, is shown in figure 11. The ratio of the lengths and the ratio of the widths of these paths are both inversely proportional to the ratio of the satellite masses. The distance of closest approach y, which is insensitive to the ratio of the satellite masses if this ratio is small ($\lesssim 0.2$), is given by (see equations (20) and (21))

$$y \approx 16\,000 \left(\frac{m_1/M}{4 \times 10^{-9}}\right) \left(\frac{0.25°/\text{day}}{\Delta n}\right)^2 \text{kilometres,} \tag{25}$$

where Δn (*ca.* 0.25°/day (Smith *et al.* 1980)) is the observed rate of approach and m_1 is the mass of the larger satellite (S10). Clearly, the satellites always remain very well separated.

It is natural to suggest that co-orbital satellites are collision products, but I consider this to be unlikely. An inspection of figure 8 shows that the path of any object leaving the main satellite is always a high α path. These paths are highly asymmetric and quite unstable. Any object on such a path will either strike the satellite or be scattered away from it. Collisions between numerous small particles leaving the main satellite may result in the low α orbits being populated, but it is difficult to see how such a mechanism could place an object with a mass comparable with that of the main satellite on such an orbit. I consider it to be more likely that S11 ($\alpha \approx 0.02$) and the other co-orbital satellites were formed from co-orbital ring particles that the main satellite failed to accrete.

It is important to realize that in a well ordered, optically thick, near-monolayer of particles, accretion is a self-limiting process. As soon as a satellite starts to form, those co-orbital particles with $\alpha \leqslant 0.2$ become inaccessible to it and those particles that are close but not co-orbital are scattered away. A satellite of diameter as small as, say, *ca.* 1 km would soon impose order on its near neighbours. The satellite could maintain a ring of co-orbital particles of width *ca.* 1 km on either side of which would be a clear, and substantially wider, gap. If such satellites exist in Saturn's rings, and the multi-ring structure suggests that they do, then these satellites will stabilize the rings against the effects of both Poynting–Robertson light drag and large-scale collisional spreading.

Description of plate 1

Figure 12. This computer-assembled two-image mosaic of Saturn's rings, taken by N.A.S.A.'s Voyager 1 on 6 November 1980 at a range of 8×10^6 km, shows approximately 95 individual ringlets. The ring structure, once thought to be produced by the gravitational interaction between Saturn's satellites and the orbits of the ring particles, now appears to be too complex for this explanation alone. The 14th satellite of Saturn, discovered by Voyager 1, is seen (lower left) just inside the narrow F ring. Image courtesy of J.P.L. and N.A.S.A.

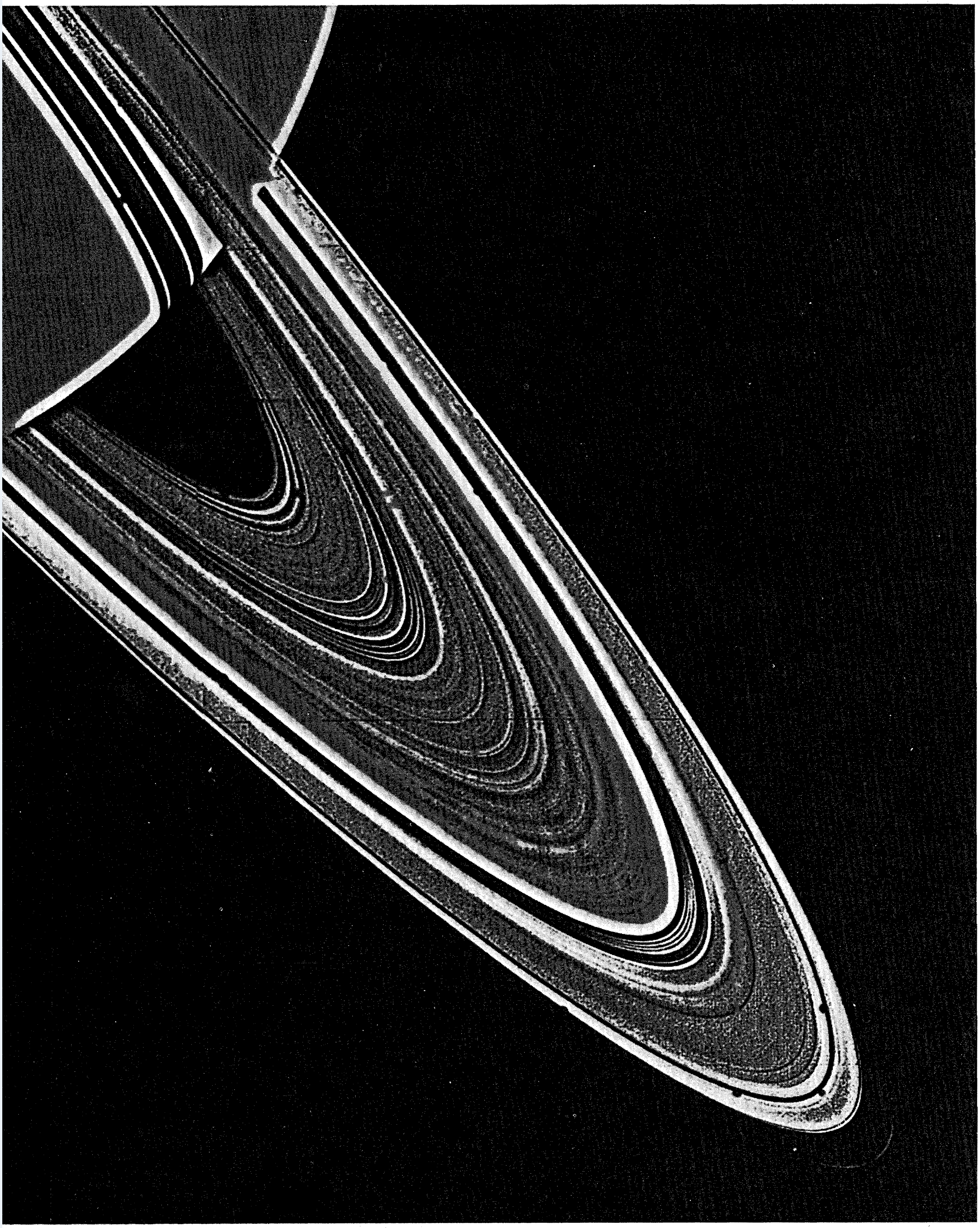


FIGURE 12. For description see opposite.

(Facing p. 274)

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

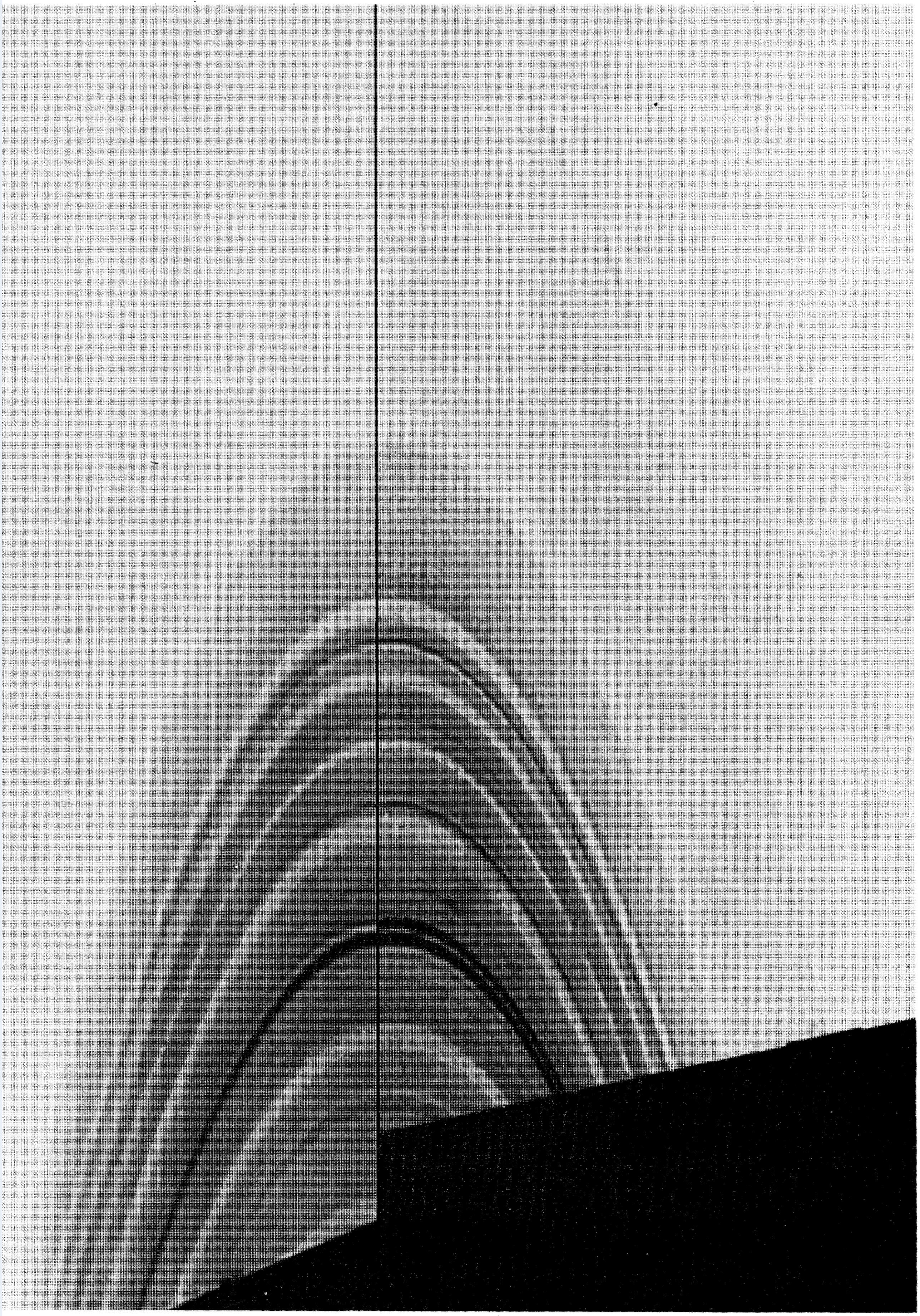


FIGURE 13. For description see opposite.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

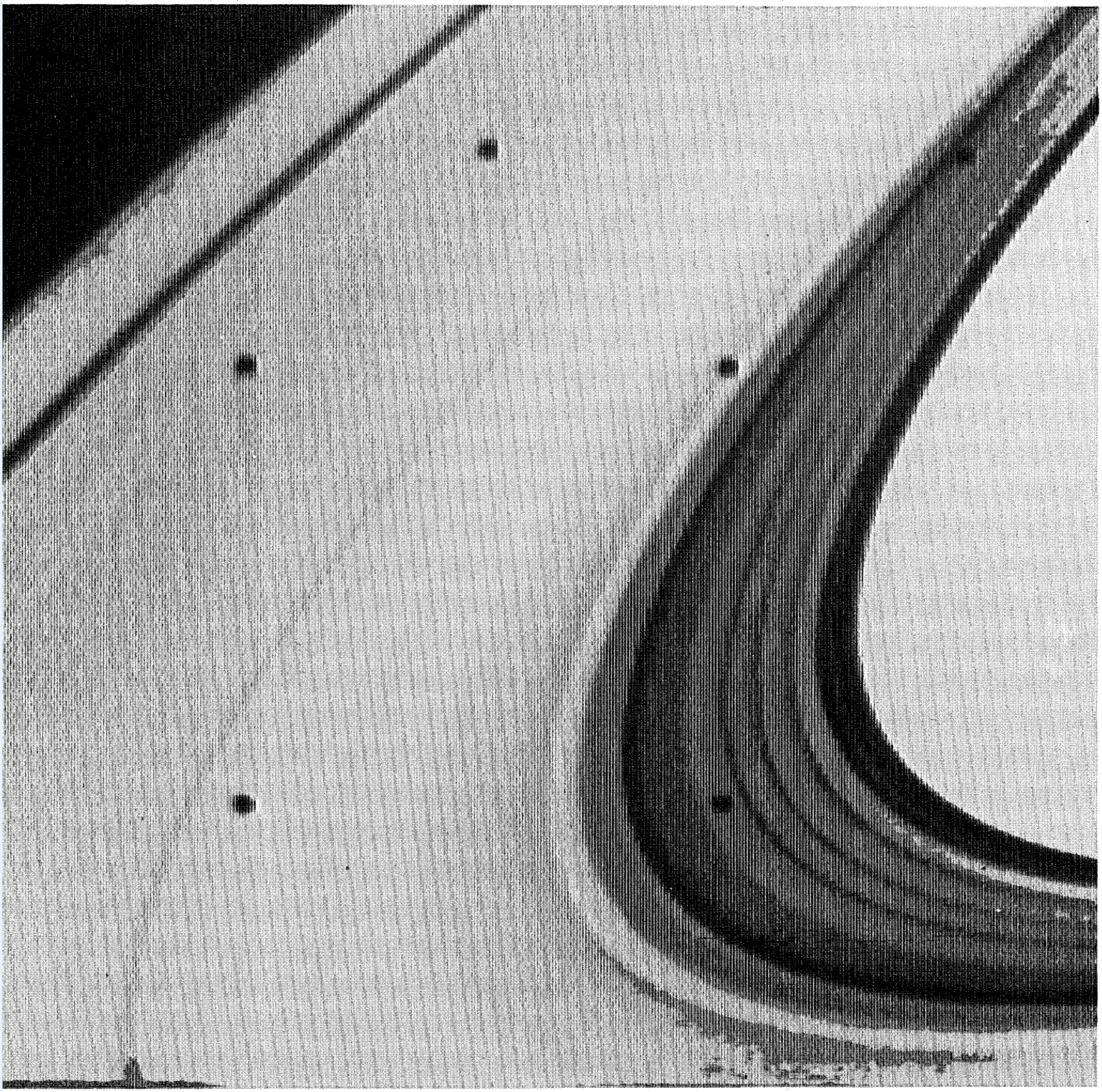


FIGURE 14. The Cassini division in Saturn's rings was imaged by Voyager 1 on 8 November 1980 at a distance of 6×10^6 km. This prominent feature in the ring system, discovered by Cassini in 1675, is defined as the region between the two dark gaps. Within the Cassini division can now be seen a number of individual features (from its outer boundary): a medium dark ringlet, 800 km wide; four brighter ringlets, approximately 500 km wide and separated by dark divisions; and a new, barely visible, narrow bright ringlet at the inner boundary which is eccentric. Image courtsey of J.P.L. and N.A.S.A.

FIGURE 13. High resolution detail in Saturn's C ring is illustrated in this composite image obtained on 10 November 1980 by Voyager 1 from a range of 3×10^6 km. The horizontal line through the centre marks the border between the two images; at the top is shown the trailing ansa of the rings, and at bottom the leading ansa. The dark gap in the centre of both images contains a narrow ring which is clearly both eccentric and of variable width. Image courtsey of J.P.L. and N.A.S.A.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

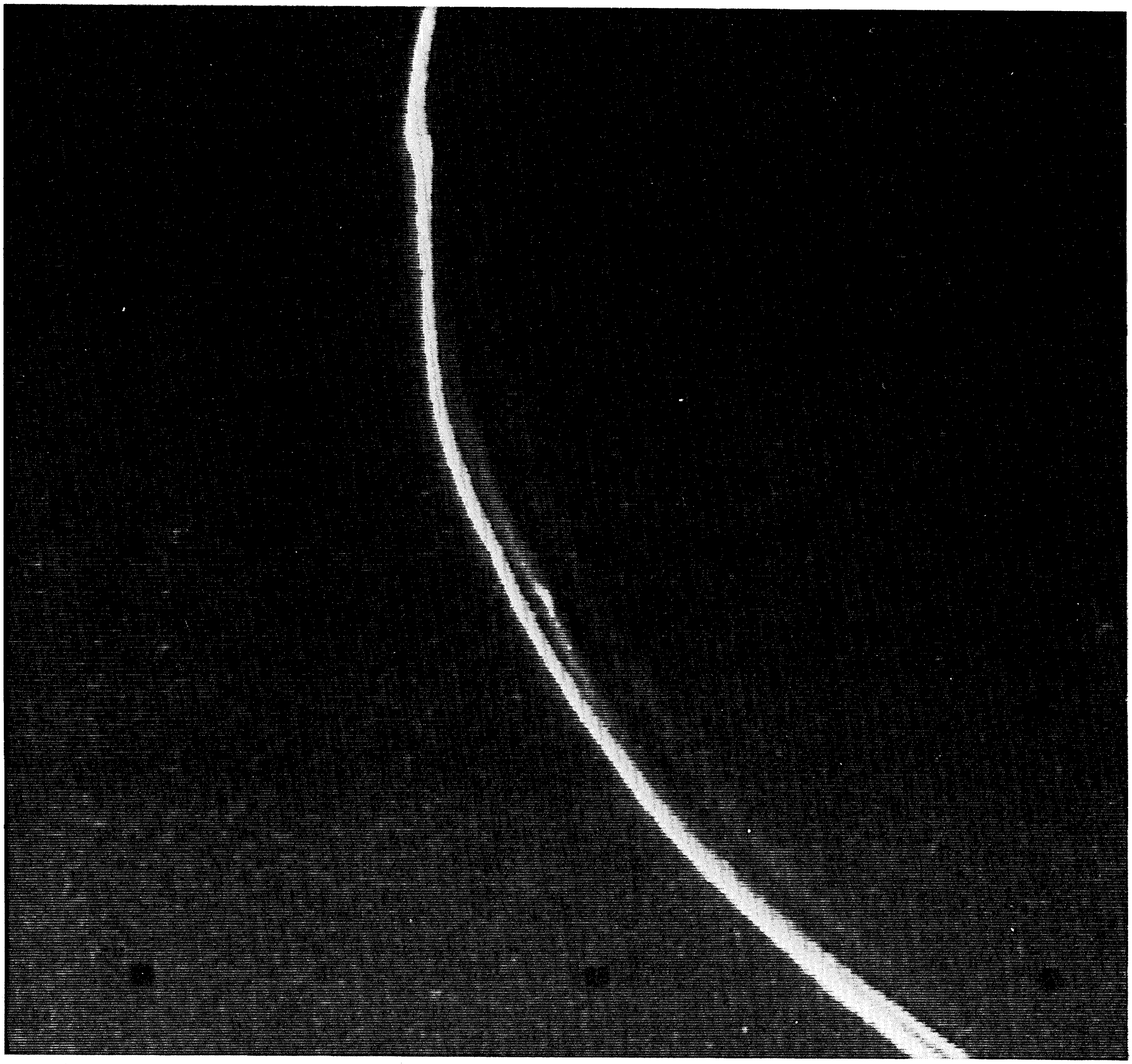


FIGURE 15. This image, taken at a range of 750000 km by Voyager 1, shows the highly complex structure of Saturn's F ring. Two narrow, 'braided', bright rings that trace distinct orbits are evident. Visible is a broader, very diffuse component, about 35 km in width. Also seen are 'knots', which probably are local clumps of ring material, but may be small satellites. Image courtesy of J.P.L. and N.A.S.A.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

6. Apse alignment

Several types of eccentric rings are dynamically feasible. For example, ring particles close to a 2:1 resonance with some satellite would be forced to move in eccentric orbits, but, as we have already stated, their orbits would not be aligned, rather $\tilde{\omega}$, $\dot{\tilde{\omega}}$ and the mean longitudes would be such that conjunctions of the particles and the satellite always occur at the apses of the orbits. Thus, the path of the particles in a frame corotating with the perturbing satellite would be a centred, that is, a non-Keplerian ellipse symmetrically disposed with respect to the satellite (see, for example, the outer path shown in figure 6). Narrow elliptical rings of this nature probably have uniform widths. The eccentric ring in Cassini's division (see figure 14), which appears to lie just outside the 2:1 Mimas resonance, may be of this kind.

However, the eccentric rings in the Solar System that are wide enough to be resolved, and whose geometry is well determined, exhibit large variations in width, increasing from a minimum at pericentre to a maximum at apocentre, and these rings have precisely aligned pericentres. They are the α, β and ε rings of Uranus and a narrow ring in Saturn's C ring (see figure 13).

The variation in width of these rings implies that there is a variation in eccentricity across the ring (Nicholson *et al.* 1978). If we define the mean eccentricity gradient g by

$$g = a\,\Delta e/\Delta a, \tag{26}$$

where $2\Delta e$ and $2\Delta a$ are the differences in the eccentricities and the semi-major axes of the Keplerian orbits that define the inner and outer edges of the ring, then the variation of radial ring width W with the true anomaly f is given by

$$W/2\Delta a = 1-(g+e)\cos f. \tag{27}$$

Since, if $e \ll 1$, the orbital radius r varies as

$$r/a = 1-e\cos f, \tag{28}$$

it follows that the harmonic variations of W and r are in phase. Consequently, W varies linearly with r (see figure 4). Since W cannot be negative, we must have $|g+e| < 1$. However, it is interesting, and probably significant, that all the observed values of g are positive and are close to the critical value of unity (see table 3). The origin of these large, positive eccentricity gradients is a matter of dispute.

Goldreich & Tremaine (1979*a*, *b*) consider that the apse alignment of narrow, eccentric rings is maintained by the self-gravitation of the ring particles. If the eccentricity gradient is positive, then the radial force exerted on a ring particle by the gravitational field of the other particles is strongest at pericentre where the ring is narrowest. This force acts against that due to the oblateness of the planet, and if g has the approximate value

$$+\frac{21}{8}\pi\left(\frac{MB^2J_2}{a^4}\right)\frac{e\langle W\rangle^2}{m_r}, \tag{29}$$

where B is the radius of the planet and J_2 the dynamical oblateness, then the ring will precess as a whole.

Dermott & Murray (1981), however, have argued that this theory does not explain why all the observed values of g are close to unity. If g is determined by three, presumably independent, parameters of the ring (e, $\langle W\rangle$ and m_r), then it is unreasonable to expect these quantities to be always such that $g \approx 0.5$. Dermott & Murray consider that the ring particles are close-packed at pericentre (see figure 7) and that it is the close-packing of the particles that prevents differential precession. They have described how differential precession, particle collisions, and self-gravitation, acting together, will always transform a narrow, eccentric ring of uniform width into a ring with a large positive eccentricity gradient.

This dispute may seem somewhat removed from the problem of the origin of the rings; however, if Goldreich & Tremaine are right, then one can use the observed value of g to determine the mass of a ring and hence the size of the ring particles. For the Uranian rings, there is, at present, no other way of doing this.

7. The Roche Zone

Since we know that the comparatively narrow Jovian ring is associated with small satellites and that these satellites are probably the source of the ring particles, and since theoretical arguments indicate strongly that the extremely narrow Uranian rings must also be associated with small satellites, it follows that the problem of the origin of these rings reduces, ultimately, to the problem of the origin of the satellites.

The coexistence of satellites and particles at the same distance from a planet forces us to reconsider the definition of Roche's limit. The radius of the Roche limit, a_L, appropriate to satellite formation is given by

$$a_L(\text{hydrostatic}) = 2.46(\rho/\rho_s)^{\frac{1}{3}} B, \tag{30}$$

where ρ and ρ_s are the mean densities of the planet and the satellite respectively. This limit is actually the limiting orbital radius at which the surface of a homogeneous, fluid satellite can remain in hydrostatic equilibrium (Chandrasekhar 1969). Kuiper (1951) defines Roche's limit for satellite formation somewhat differently, with the result that the coefficient 2.46 becomes 2.52, but for the purposes of this paper this difference is ignorable.

The distance at which a satellite would suffer tidal loss of particles or disruption depends upon the properties of the material and the shape of the satellite, as well as on its density. If the material can flow to adopt the hydrostatic equilibrium shape, then the critical distance is simply a_L (hydrostatic). Outside this distance the velocity of escape at all points on the satellite surface is non-zero and all loose particles are retained. However, for a body that does not flow towards the equilibrium shape, other effective values of Roche's limit apply. In particular, outside the limit,

$$a_L(\text{spherical}) = (3\rho/\rho_s)^{\frac{1}{3}} B; \tag{31}$$

a perfectly spherical satellite could retain loose particles on all points of its surface. Dermott *et al.* (1979) refer to the range of values of a_L between the above limits as the Roche zone. It is in this zone that a satellite of any shape would be expected to shed loose material. The narrow orbital radii range of the Jovian and Uranian rings in fact fits well the Roche zones that would be defined for a likely range of density and shape of the satellites (see figure 1).

Dermott *et al.* (1979) and Dermott *et al.* (1980) point out that the radii of the synchronous orbits of Uranus and Jupiter are appreciably greater than Roche's limit for satellite formation (see figure 1). Consequently, satellites formed outside Roche's limit for satellite formation but inside the synchronous orbit could have slowly spiralled in towards the planet under the action of tidal forces. They would have then entered the Roche zone, in which part of their surfaces would have been subject to an outward force, and in those areas a tensile strength would have been required to prevent loss of material. If the impact histories of these satellites were as violent and destructive as the shapes of some of the remaining small satellites, Phobos, Amalthea and S10 for example, suggest, then these satellites were probably already partly fragmented before entering the Roche zone and thus very large chunks, many kilometres in size, as well as dust and debris, may have been lost from the main satellite body. We consider that these chunks, which are too small to have been removed by tidal forces, have acted to organize the associated dust and debris into the present systems of narrow rings.

A lower limit to the mass of a satellite that could suffer orbital decay due to tidal dissipation is given by

$$m = (a_1/a_x)^{\frac{13}{2}} m_x, \tag{32}$$

where a_1 is the initial orbital radius of the satellite and the suffix x refers to Io in the Jovian system (here, I neglect that Io is part of a stable resonance) and to Ariel in the Uranian system (Dermott *et al.* 1979). In both systems m must be $\gtrsim 10^{-8} M$, and in the Jovian case this corresponds to $m \gtrsim 2 \times 10^{22}$ g. This is, of course, very much greater than the present ring mass (*ca.* 10^{20} g (Burns *et al.* 1981)). However, this is not a problem if it is allowed that the main satellite, of diameter *ca.* 270 km, was not disrupted by tides but spiralled into the planet.

A satellite of radius r_s could orbit at the surface of a planet if it had a tensile strength

$$T \geqslant \tfrac{8}{57}\pi G\rho\rho_s r_s^2 \tag{33}$$

(Aggarwal & Oberbeck 1974). Thus, if $r_s \approx 135$ km, then T must be $\geqslant 2 \times 10^7$ dyn cm^{-2}. This estimate makes many assumptions about the mode of fracture, the shape and the history of the satellite and it may well be that a satellite of material strength greater than the above limit would break up before reaching the planet surface. However, this might happen at such a close distance to the planet that tidal forces could still remove the fragments. This requires further investigation.

8. Discussion

The existence of satellites in stable, horseshoe orbits lends support to the horsehoe orbit ring model. However, other observations argue against it. In particular, recent occultation data show that the structure of the Uranian η ring is highly complex (see figure 16). Such asymmetry could not possibly be modelled by a ring that contains a single satellite: one would have to suppose that the ring contains a number of small satellites. But some of the other rings whose profiles have been resolved (the α and ε Uranian rings) are also somewhat asymmetric and thus these also argue against the model. Whether satellites exist in Saturn's rings that maintain rings of particles on horseshoe paths is not known, but I would argue that they probably do. I also consider that horseshoe orbits had a role in the formation of the observed co-orbital satellites.

The existence of satellites either side of Saturn's F ring would appear to support the Goldreich & Tremaine theory of ring formation. However, I would not regard their theory as confirmed until it has been shown that the masses and the orbital configurations of the satellites are such that the mean tidal torques that they exert on the ring are equal and opposite (see equation (13)). Also, we need evidence that the characteristic size of the ring particles is consistent with the observed ring width (see equation (12)).

We must also consider that other small Saturnian satellites have been discovered. S15 orbits just outside the A ring, but no narrow ring is observed between this satellite and its nearest neighbour, S14, which orbits just inside the F ring. Furthermore, a narrow ring, the G ring (width < 300 km: the resolution is limited by smear), has been observed by Voyager 1 at 2.8 planetary radii. Since no satellites have been observed in its vicinity and since the masses and the orbital radii of its apparent nearest neighbours, Mimas and the co-orbital satellites S10 and S11, do not

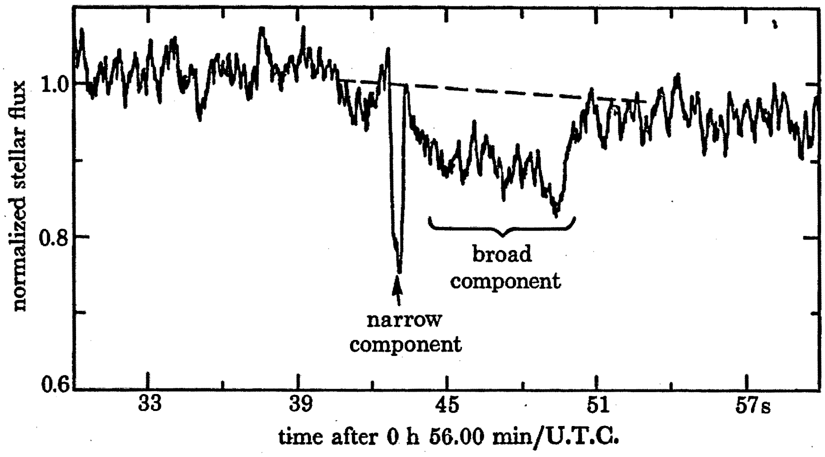


FIGURE 16. Occultation profile of the Uranian η ring (Elliot *et al.* 1981 *b*). The narrow component is unresolved and lies at the inner edge of the broad components, which is about 60 km wide.

satisfy the tidal torque criterion (equation (13)), it would be of interest to know if this ring is sufficiently narrow that it must be regarded as confined.

Most of the narrow Uranian rings (the η ring may be an exception (see figure 16)) do not appear to exhibit structure as complex as that of the saturnian F ring. The occultation profiles of the Uranian ε ring (see figure 3), for example, are always similar even though their widths are different and thus the profiles refer to different parts of the ring. Occultation profiles of the Saturnian F ring (see figure 15, plate 4) could not possibly show this regularity. However, this may not pose a problem for the Goldreich & Tremaine theory. The masses of the confining satellites in the Uranian system may be such that the gravitational perturbations of the ring particle orbits (see equations (8) and (9)) are less than the resolution achieved by present-day occultation techniques.

In conclusion, studies of the dynamics of planetary ring systems show promise of revealing some of the accretion mechanisms that operated in the early Solar System. It seems likely that horseshoe orbits, spiral density waves and tidal torques exerted on disks of particles by nearby satellites all had a role in the development of the present structure and therein lies the importance of these ring studies. The provenance of the ring material is more problematical. I consider that the narrow Jovian and Uranian ring systems (particles plus satellites) are the results of the break-up of satellites which formed below synchronous height and then spiralled in towards the planets as a result of tidal dissipation within the planets. The broad Saturnian system could not have been formed in this way and we have little to say about the formation of this system. It would be of interest to know if there are any compositional variations within the ring and if small satellites (diameters $\gtrsim$ 1 km) exist within the multitudinous narrow gaps.

This research was supported by N.S.F. Grant AST 79164-74-A01.

REFERENCES (Dermott)

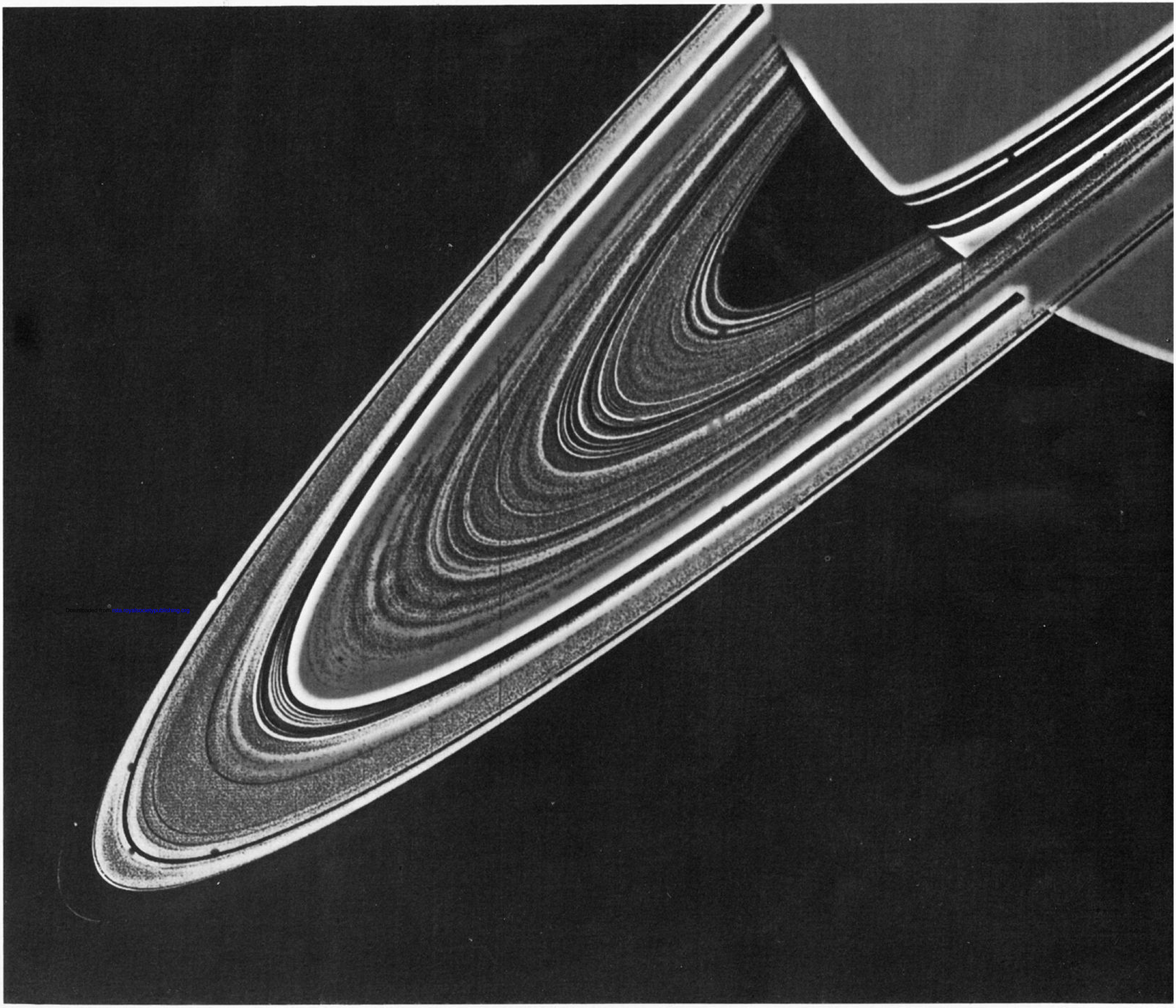
Aggarwal, H. R. & Oberbeck, V. R. 1974 *Astrophys. J.* **191**, 577–588.

Baum, W. A., Kreidl, T., Westphal, J. A., Danielson, G. E., Seidelmann, P. K., Pascu, D & Currie, D. G. 1980 *Bull. Am. astr. Soc.* **12**, 700–701.

Berge, G. L. & Muhleman, D. O. 1973 *Astrophys. J.* **185**, 373–381.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Brahic, A. 1977 *Astron. Astrophys.* **54**, 895–907.
Brahic, A., Lecacheux, J. & Sicardy, B. 1980 *Bull. Am. astr. Soc.* **12**, 700.
Briggs, F. H. 1974 *Astrophys. J.* **189**, 367–377.
Brouwer, D. & Clemence, G. M. 1961 In *Celestial mechanics*, p. 337. New York and London: Academic Press.
Brown, E. W. 1911 *Mon. Not. R. astr. Soc.* **71**, 438–454.
Burns, J. A., Showalter, M. R., Cuzzi, J. N. & Pollack, J. B. 1981 *Icarus, N.Y.* **44**, 339–360.
Chandrasekhar, S. 1969 *Ellipsoidal figures of equilibrium.* New Haven: Yale University Press.
Dermott, S. F. 1981 *Nature, Lond.* **290**, 454–457.
Dermott, S. F., Gold, T. & Sinclair, A. T. 1979 *Astr. J.* **84**, 1225–1234.
Dermott, S. F. & Murray, C. D. 1980 *Icarus, N.Y.* **43**, 338–349.
Dermott, S. F. & Murray, C. D. 1981 *Icarus, N.Y.* (Submitted for publication.)
Dermott, S. F., Murray, C. D. & Sinclair, A. T. 1980 *Nature, Lond.* **284**, 309–313.
Dilley, J. 1980 *Bull. Am. astr. Soc.* **12**, 699.
Dollfus, A. & Brunier, S. 1980 *C.r. hebd. Séanc. Acad. Sci. Paris* B **290**, 261–263.
Elliot, J. L., Dunham, E. & Mink, D. 1977 *Nature, Lond.* **267**, 328–330.
Elliot, J. L., French, R. G., Frogel, J. A., Elias, J. H., Mink, D. & Liller, W. 1981*a* *Astr. J.* **86**, 444–455.
Elliot, J. L., Frogel, J. A., Elias, J. H., Glass, I. S., French, R. G., Mink, D. & Liller, W. 1981*b* *Astr. J.* **86**, 127–134.
Feibelman, W. A. 1967 *Nature, Lond.* **214**, 793–794.
Gehrels, T., Baker, L. R., Beshore, E., Bleman, C., Burke, J. J., Castillo, N. D., Dacosta, B., Degewij, J., Doose, L. R., Fountain, J. W., Gotobed, G., Kenknight, C. E., Kingston, R., McLaughlin, G., McMillan, R., Murphy, R., Smith, P. H., Stoll, C. P., Strickland, R. N., Tomasko, M. G., Wijesinghe, M. P., Coffeen, D. L., & Esposito, L. 1980 *Science, N.Y.* **207**, 434–439.
Goldreich, P. 1979 In *Dynamics of the Solar System* (ed. R. L. Duncome), pp. 191–196. Dordrecht: Reidel.
Goldreich, P. & Tremaine, S. 1978*a* *Icarus, N.Y.* **34**, 227–239.
Goldreich, P. & Tremaine, S. 1978*b* *Icarus, N.Y.* **34**, 240–253.
Goldreich, P. & Tremaine, S. 1979*a* *Nature, Lond.* **277**, 97–99.
Goldreich, P. & Tremaine, S. 1979*b* *Astr. J.* **84**, 1638–1641.
Goldstein, R. M. & Morris, G. A. 1973 *Icarus, N.Y.* **20**, 260–278.
Greenberg, R. 1973 *Astr. J.* **78**, 338–346.
Jewitt, D. C. & Danielson, G. E. 1981 *J. geophys Res.* (In the press.)
Kuiper, G. P. 1951 In *Astrophysics* (ed. J. A. Hynek), ch. 8. New York: McGraw-Hill.
Lamy, P. L. & Mauron, N. 1980 *Bull. Am. astr. Soc.* **12**, 728.
Lecacheux, J., Laques, P., Vapillon, L., Auge, A. & Despiau, R. 1980 *Icarus, N.Y.* **43**, 111–115.
Lin, D. N. C. & Papaloizou, J. 1979 *Mon. Not. R. astr. Soc.* **186**, 799–812.
Millis, R. L., Wasserman, L. H. & Birch, P. V. 1977 *Nature, Lond.* **267**, 330–332.
Nicholson, P. D., Persson, S. E., Matthews, K., Goldreich, P. & Neugebauer, G. 1978 *Astr. J.* **83**, 1240–1248.
Pilcher, C. B., Chapman, C. R., Lebofsky, L. A. & Kieffer, H. H. 1970 *Science, N.Y.* **167**, 1372–1373.
Reitsema, H. J., Smith, B. A., Larson, S. M. 1980*a* *Bull. Am. astr. Soc.* **12**, 701.
Reitsema, H. J., Smith, B. A. & Larson, S. M. 1980*b* *Icarus, N.Y.* **43**, 116–119.
Smith, B. A. 1977 *Nature, Lond.* **268**, 32.
Smith, B. A., Reitsema, H. J., Fountain, J. W. & Larson, S. M. 1980 *Bull. Am. astr. Soc.* **12**, 727.
Terrile, R. J. & Tokanaga, A. 1980 *Bull. Am. astr. Soc.* **12**, 701.
Van Allen, J. A., Thomsen, M. F., Randall, B. A., Rairden, R. L. & Grosskreutz, C. L. 1980 *Science, N.Y.* **207**, 415–421.



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

FIGURE 12. This computer-assembled two-image mosaic of Saturn's rings, taken by N.A.S.A.'s Voyager 1 on 6 November 1980 at a range of 8×10^6 km, shows approximately 95 individual ringlets. The ring structure, once thought to be produced by the gravitational interaction between Saturn's satellites and the orbits of the ring particles, now appears to be too complex for this explanation alone. The 14th satellite of Saturn, discovered by Voyager 1, is seen (lower left) just inside the narrow F ring. Image courtesy of J.P.L. and N.A.S.A.

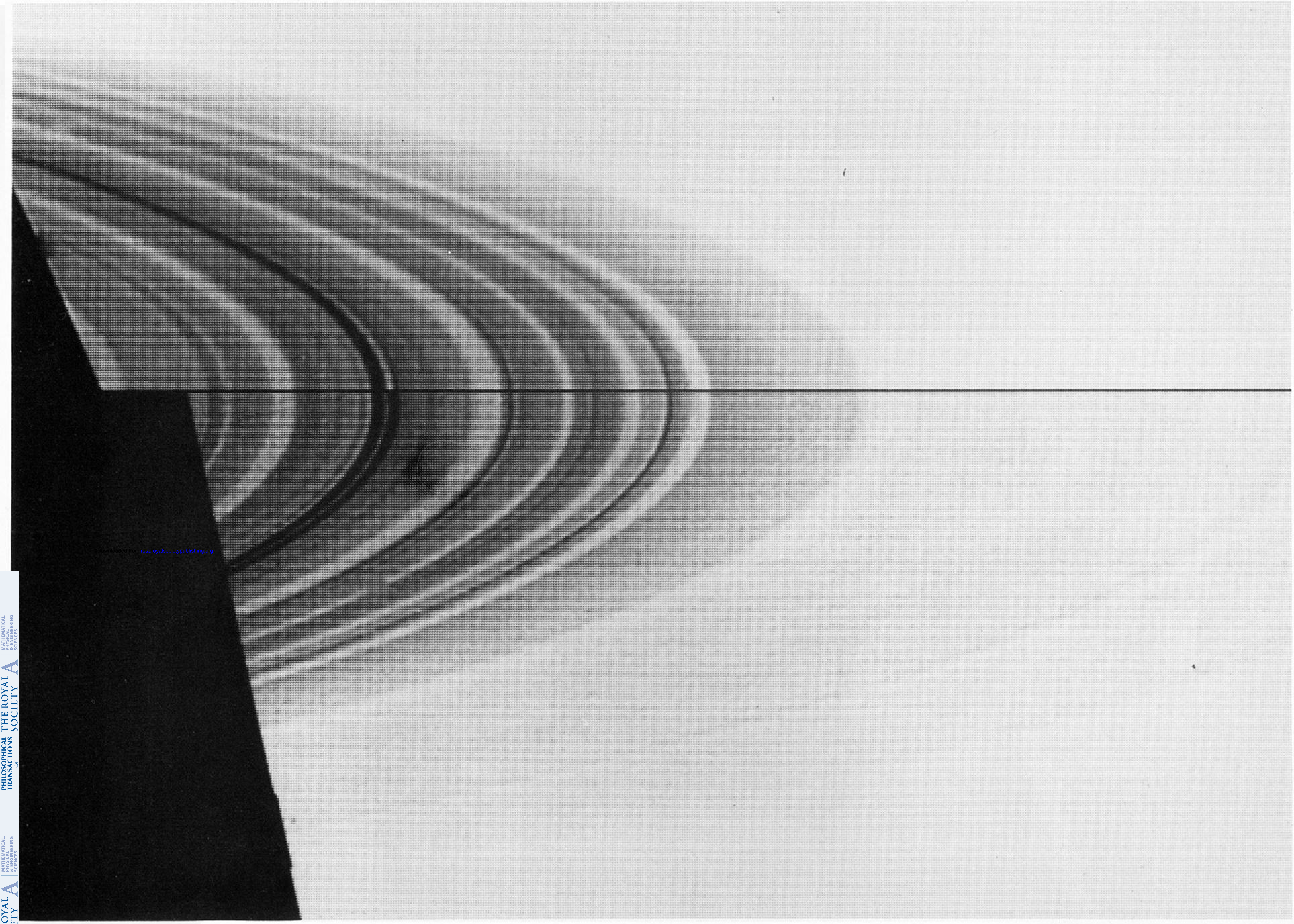


FIGURE 13. High resolution detail in Saturn's C ring is illustrated in this composite image obtained on 10 November 1980 by Voyager 1 from a range of 3×10^6 km. The horizontal line through the centre marks the border between the two images; at the top is shown the trailing ansa of the rings, and at bottom the leading ansa. The dark gap in the centre of both images contains a narrow ring which is clearly both eccentric and of variable width. Image courtsey of J.P.L. and N.A.S.A.

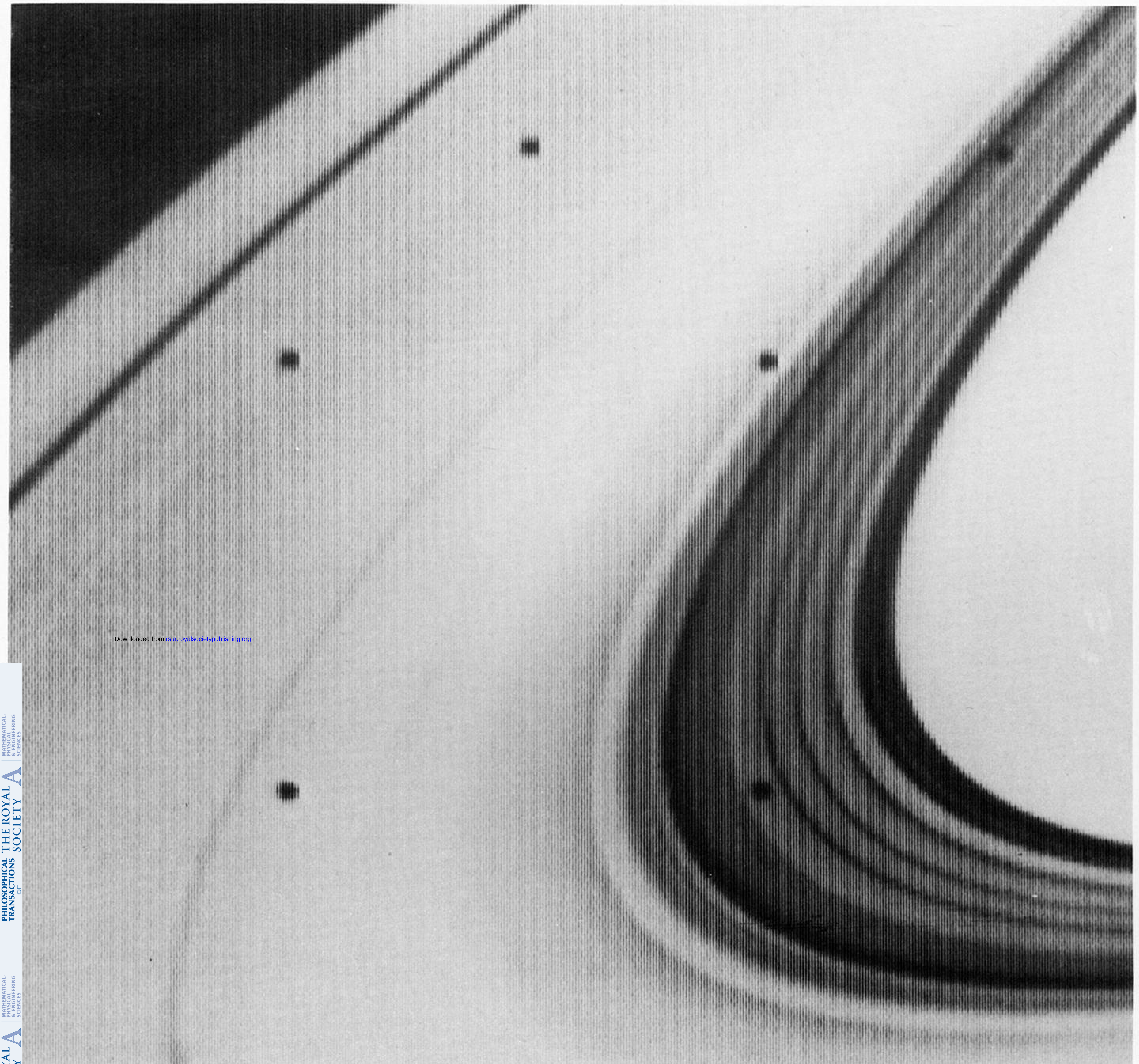
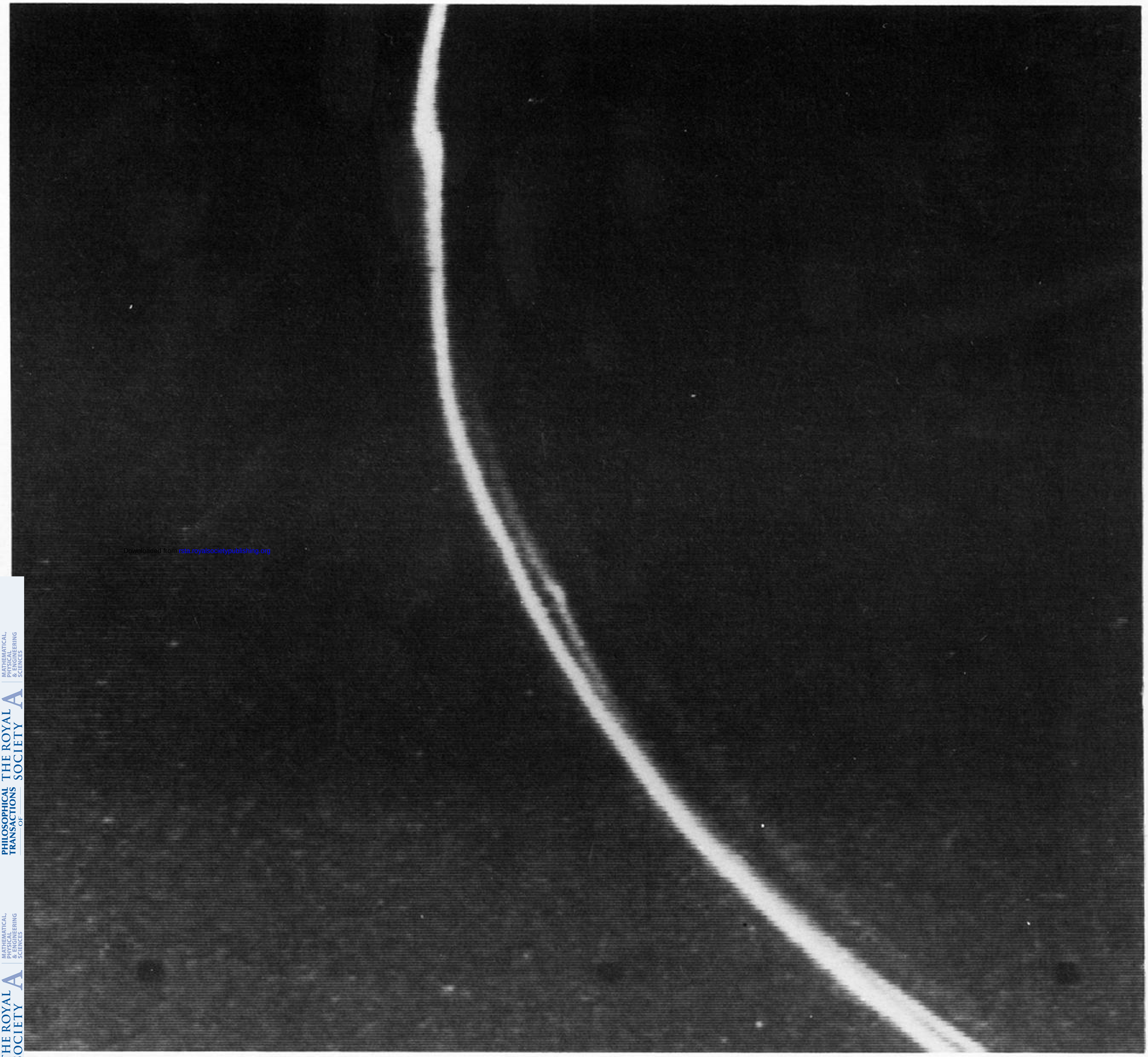


FIGURE 14. The Cassini division in Saturn's rings was imaged by Voyager 1 on 8 November 1980 at a distance of 6×10^6 km. This prominent feature in the ring system, discovered by Cassini in 1675, is defined as the region between the two dark gaps. Within the Cassini division can now be seen a number of individual features (from its outer boundary): a medium dark ringlet, 800 km wide; four brighter ringlets, approximately 500 km wide and separated by dark divisions; and a new, barely visible, narrow bright ringlet at the inner boundary which is eccentric. Image courtesy of J.P.L. and N.A.S.A.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES



PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

FIGURE 15. This image, taken at a range of 750 000 km by Voyager 1, shows the highly complex structure of Saturn's F ring. Two narrow, 'braided', bright rings that trace distinct orbits are evident. Visible is a broader, very diffuse component, about 35 km in width. Also seen are 'knots', which probably are local clumps of ring material, but may be small satellites. Image courtesy of J.P.L. and N.A.S.A.